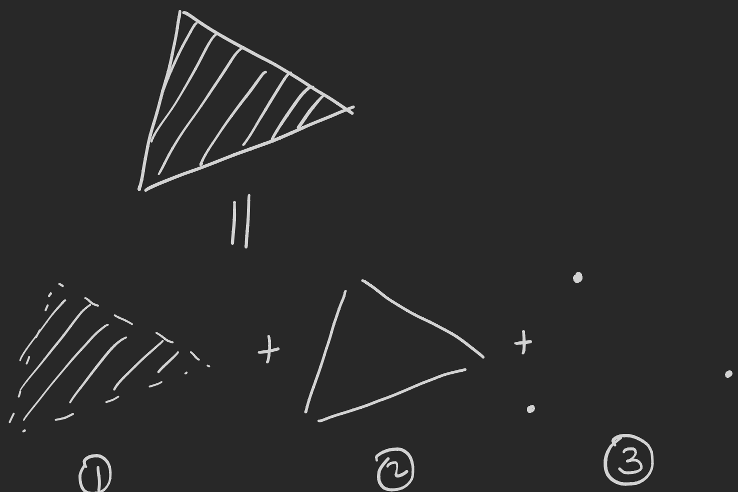
§14.7 #37:

Find extrema of $f(x,y) = 2x^3 + y^4$

on $D = \{(x,y) \mid x^2+y^2 \le 1\}$

If region over which we are optimizing looks like...



for ①, just find critical pts: $\nabla f = \vec{0}$.

These are the candidates for extrema.

② For each line segment, reduce to a SVC problem by rewriting $f(x,y)$ as a function of a single var.

③ Each isolated point is itself a candidate.

In our problem:

$D =$ 

$x^2+y^2 \le 1 \qquad x^2+y^2<1 \qquad x^2+y^2=1$

① $\nabla f(x,y) = \langle 6x^2, 4y^3 \rangle = \vec{0}$

only sol is $x=0 \quad y=0$

(is in $x^2+y^2<1$ ✓)

② $2x^3+y^4 = 2x^3+(1-x^2)^2$

using $x^2+y^2=1$

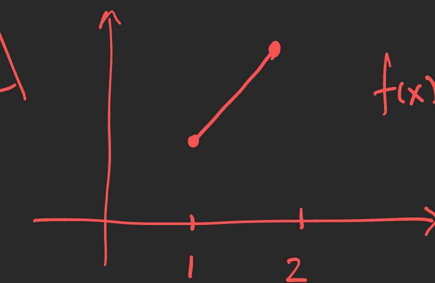
$= 2x^3+1-2x^2+x^4$

$= x^4+2x^3-2x^2+1$

$g(x) = x^4+2x^3-2x^2+1$

Then look for when $g'(x)=0$.

⚠

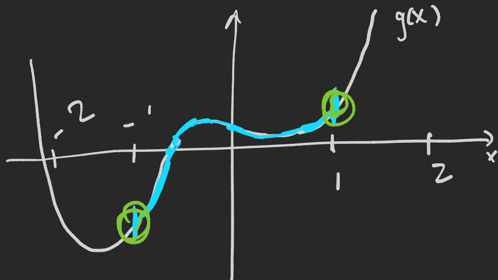


$f(x)=x$ on domain $[1,2]$

The min/max are obvious @ endpoints, but calculus (i.e. $f'(x)=0$) doesn't see them.

Why is ⚠ relevant to this question?

b/c in ②, x is constrained to the interval $[-1,1]$ and doing $g'(x)=0$ will only find candidates strictly between -1 and 1.



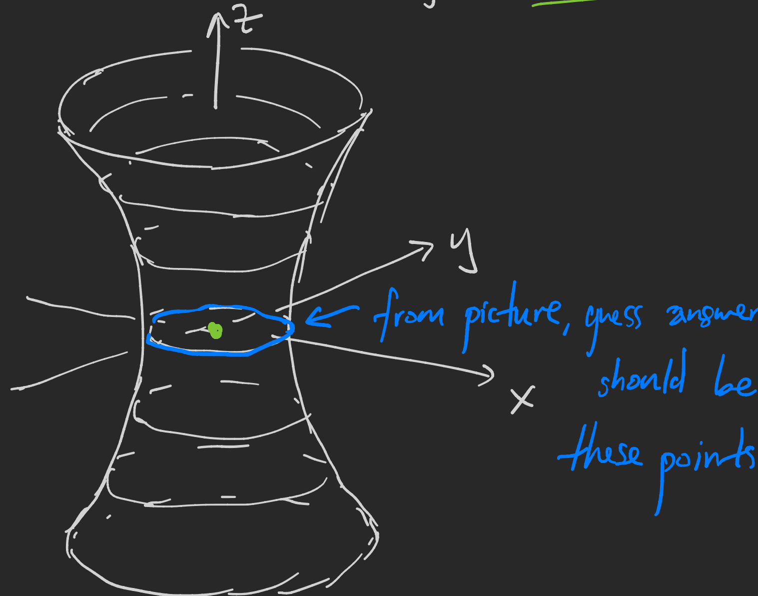
So really we also have to consider the two points $x=-1$ and 1 as candidates separately.

Another example:

What point(s) on the surface

$$x^2+y^2-z^2=1$$

are closest to the origin $(0,0,0)$?



Algebraically, want to minimize

$$\sqrt{x^2+y^2+z^2}.$$

Equivalent to minimizing $x^2+y^2+z^2$.

$$x^2+y^2+z^2 = 2x^2+2y^2-1$$

$$x^2+y^2-z^2=1$$

Call $f(x,y) = 2x^2+2y^2-1$.

$$\nabla f(x,y) = \langle 4x, 4y \rangle = \vec{0}$$

Only solution: $(0,0)$

⚠ What went wrong: I am <u>not</u> trying to minimize $f(x,y)=2x^2+2y^2-1$ on <u>all</u> of $\mathbb{R}^2$; rather, only on

$x^2+y^2-1 \geq 0$

① is analyzed by $\nabla f = \vec{0}$, which is what I did

② needs to be considered separately, which I did not do.

Bottom line: when eliminating variables, be careful not to forget constraints! New "boundary" regions might appear.

Suppose that f is a function defined on all of $\mathbb{R}^2$. If $f_x(x, y) = 3xy + y^2$, which of the following *cannot* be $f_y(x, y)$?

$3x^2/2 + 2y(x + y^2)$	**A**
$3xy + x^2$	**B**
$3x^2/2 + 2xy$	**C**
All of the above are possible	**D**

Method 1:

Clairaut's Thm says:

$$(f_x)_y = (f_y)_x$$

$$\frac{\partial}{\partial y}(3xy+y^2) = (f_y)_x$$

$$\| $$

$$3x+2y$$

If $f_y(x,y) = 3xy + x^2$, then

$$f_{yx}(x,y) = 3y+2x \neq 3x+2y$$

so this is impossible.

Method 2: Just integrate.

$$f_x(x,y) = 3xy + y^2$$

$$f(x,y) = \frac{3}{2}x^2y + y^2x + \underbrace{C(y)}$$

some function of y alone (i.e. a "constant" from x's perspective)

$$f_y(x,y) = \frac{3}{2}x^2 + 2yx + C'(y).$$

$$\neq 3xy + x^2$$

(but the other answer choices are possible.)

Compute $\lim_{(x,y)\to(0,0)} \frac{xy\cos(y)}{3x^2+y^2}$ **if it exists (or write "DNE" if not).**

Compute $\lim_{(x,y)\to(0,0)} \dfrac{\sin^2(\sqrt{x^2+y^2})}{x^2+y^2}$ **if it exists (or write"DNE" if not).**

#1) Along $y=mx$:

$$\lim_{x\to 0} \frac{mx^2\cos(mx)}{3x^2+m^2x^2}$$

$$= \lim_{x\to 0} \frac{m}{3+m^2}\cos(mx)$$

$$= \frac{m}{3+m^2}.$$ This depends on m, so limit DNE.

#2) $$\lim_{\substack{r\to 0^+ \\ \theta \text{ any}}} \frac{\sin^2(r)}{r^2}$$

$$= \left(\lim_{\substack{r\to 0^+ \\ \theta \text{ any}}} \frac{\sin(r)}{r}\right)^2 = 1^2 = 1$$

just a SVC limit